Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐ **Total No. of Pages: 04**

Total No. of Questions: 07

M.Sc. (Mathematics) (Sem. – 4)

OPERATIONS RESEARCH

Subject Code: MSM503-18

M Code: 77873

Date of Examination : 17-12-2022

Time: 3 Hrs. **Max. Marks: 70**

INSTRUCTIONS TO CANDIDATES:

1. **SECTION-A is COMPULSORY consisting of FIVE questions carrying TWO marks each.**
2. **SECTION-B contains THREE questions carrying FIFTEEN marks each and students have to attempt any TWO questions.**
3. **SECTION-C contains THREE questions carrying FIFTEEN marks each and students have to attempt any TWO questions.**

SECTON-A

1. Write briefly:

a) Is the union of two convex sets is convex? Justify your answer.

b) Determine the maximum/minimum (if any) of the following function:

$$f(x_1, x_2) = x_1 + 2x_3 + x_2x_3 - x_1^2 - x_2^2 - x_3^2$$

c) Solve the following linear programming problem

$$max\ Z = x_1 + 2x - 2 + x_3 + x_4$$

Subject to $$x_1 + x_2 + 3x_3 + 4x_4 = 12, x_1, x_2, x_3, x_4 \geq 0$$

d) Express the following assignment problem as a linear programming problem

	J_1	J_2	J_3
W_1	1	3	4
W_2	6	2	7
W_3	4	3	1

e) Define a convex function. Is the function $f(x) = |x+1|$ convex?

SECTION-B

2. Solve the following LPP by using Big M method (15)

$$max\ 4x_1 + 3x_2 + 5x_3$$

Subject to
$$x_1 + 3x_2 + 2x_3 \leq 10$$
$$2x_1 + 2x_2 + x_3 \geq 6$$
$$x_1 + 2x_2 + 3x_3 = 14, \quad x_1, x_2, x_3 \geq 0$$

3. a) State and prove weak duality theorem. (7)

b) Consider the following linear programming problem (8)

$$\max \quad Z = 4x_1 + 3x_2$$
$$\text{subject to,} \quad x_1 + x_2 \leq 8, 2x_1 + x_2 \leq 10, x_1, x_2 \geq 0$$

Solve this problem graphically and then using complementary slackness theorem find an optimal solution of its dual.

4. a) Consider the following linear programming problem (10)

$$\max \quad Z = 4x_1 + 6x_2 + 2x_3$$

subject to $\quad x_1 + x_2 + x_3 + x_4 = 3, \quad x_1 + 4x_2 + 7x_3 + x_5 = 9, \quad x_1, x_2, x_3, x_4, x_5 \geq 0$

An optimal table of a LPP is given below

c_B	Basis	x_1	x_2	x_3	x_4	x_5	X_B (Sol.)
	$z_j - c_j$	0	0	6	10/3	2/3	$Z = 16$
4	x_1	1	0	−1	4/3	−1/3	1
6	x_2	0	1	2	−1/3	1/3	2

i) If an additional constraint $2x_1 + 3x_2 - 2x_3 \leq 4$ is added, will the current optimal solution get disturbed? If so, find a new optimal solution.

ii) If the coefficients of x_3 in the constraints are changed from $(1,7)^T$ to $(1,2)^T$, discuss the effect of this change in the given optimal solution.

iii) What happens if the RHS of the constraints is changed from $(3,9)^T$ to $(7,17)^T$?

b) An optimal table of this problem is given below (5)

c_B	Basis	x_1	x_2	x_3	x_4	x_5	X_B (Sol.)
	$z_j - c_j$	0	0	$17/7$	$6/7$	$4/7$	$Z = 2$
4	x_2	0	1	$1/7$	$2/7$	$-1/7$	0
2	x_1	1	0	$17/7$	$-1/7$	$4/7$	1

Construct the original LPP. It is given that x_4 and x_5 are slack variables.

SECTION-C

5. a) Consider the data of a project as shown in the following table. (10)

Activity	Normal time (Weeks)	Normal cost (Rs.)	Crash time (Weeks)	Crash Cost (Rs.)
$1-2$	5	400	4	460
$1-3$	13	700	9	900
$1-4$	7	600	4	810
$3-5$	12	800	11	865
$2-3$	6	900	4	1130
$2-4$	5	1000	3	1180
$4-5$	9	1500	6	1800

i) Draw the network and find the normal duration, normal duration and critical path.

ii) Find the optimal cost for completing the project in 22 days?

b) There are four jobs A, B, C and D and these are to be performed on four machine centres I, II, III and IV. One job is to be allocated to a machine center, though each machine is capable of doing any job at different cost given by the matrix below: (5)

	A	B	C	D
I	5	7	11	6
II	8	5	9	6
III	4	7	10	7
IV	10	4	8	3

Find the allocation of jobs to the machines so that the total cost of processing is minimum

6. Consider a cost minimizing transportation problem whose cost matrix is given below: (15)

	D_1	D_2	D_3	D_4	a_i
S_1	11	13	17	14	250
S_2	16	23	14	9	300
S_3	21	24	13	10	400
b_j	200	225	275	250	

Where $a_i, i = 1,2,3$ and $b_j, j = 1,2,3,4$ is the availability and demand at source S_i and destination D_j respectively.

a) Find initial basic feasible solution using least cost method?

b) Is the solution obtained in part (i) above optimal? If not, then find an optimal feasible solution of this problem?

7. a) Use Wolfe's method to solve the following Quadratic programming problem: (10)

$$\max Z = x_1 + x_2 - x_1^2 + 2x_1x_2 - 2x_2^2$$

subject to $2x_1 + x_2 \leq 1, x_1, x_2 \geq 0$

b) Consider the nonlinear programming problem (5)

$$\min Z = -x_2$$

Subject to $x_1^2 + x_2^2 \leq 4, \quad -x_1^2 + x_2 \leq 0$

Verify that the KKT conditions are satisfied at (0,0), but it, is not. a global (not even a local) minimum point.

NOTE : Disclosure of Identity by writing Mobile No. or Marking of passing request on any paper of Answer Sheet will lead to UMC against the Student.